Technical Comments

Comment on "Temperature Distribution in a Porous Surface"

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RECENTLY Libby[1] presented an analysis of the temperature distribution in a porous surface associated with suction of a high-energy boundary-layer flow. Libby concludes that with adequate radiation cooling from the surface, suction for boundary-layer control purposes may provide a source of coolant air. Our purpose is to show that Libby's conclusion is incorrect and is indeed not substantiated by his analysis.

Libby has solved for the fluid and solid temperature distributions across a porous wall subject to the boundary conditions that at $x = 0$ (the exterior surface) the fluid temperature t_0 is specified and is equal to the solid temperature $T(0)$, and at $x = L$ the coolant temperature is specified to be t_c where $t_c < t_0$. The results presented in Figs. 2 and 3 (of Ref. 1) represent a valid solution to the problem posed, but do not substantiate assertions made by Libby, namely 1) if the radiative flux from the surface q_r exceeds the convective flux into the surface q_c, the ingested fluid is cooled in its passage through the porous wall, and 2) conduction within the solid at the interior surface, compared to the difference $(-q_c + q_r)$, can be neglected.

In order to show that these statements are incorrect we reproduce Eq. (3) of Ref. 1, which is simply the exterior surface energy balance assuming a perfectly opaque surface

$$\lambda_s T'(0^+) = q_r - q_c \tag{1}$$

and a corrected form of Eq. (4), which is the integrated energy conservation equation across the porous wall,

$$\rho v c_p(t_0 - t_c) = -q_c + q_r - \lambda_s T'(L) \tag{2}$$

It is immediately apparent from Eq. (1) that if $q_r > q_c$, then $T'(0^+)$ is positive. Since $T'(0^+)$ is negative in Figs. 2 and 3, the solutions presented by Libby do not correspond to $q_r > q_c$ but rather to $q_r < q_c$. Conduction in the solid at $x = L$ can be related to conduction at $x = 0$ by combining Eqs. (1) and (2) to yield

$$-\lambda_s T'(L) = -\lambda_s T'(0^+) + \rho v c_p(t_0 - t_c) \tag{3}$$

Thus for the situation considered by Libby, the conduction in the solid at $x = L$ is larger than the conduction at $x = 0$, a result which is verified by Figs. 2 and 3 of Ref. 1. Recall from Eq. (1), $\lambda_s T'(0^+) = q_r - q_c$, and hence not only can $\lambda_s T'(L)$ not be neglected compared with $(-q_c + q_r)$, but the absolute magnitude of $\lambda_s T'(L)$ is greater than that of $(-q_c + q_r)$. It follows that the ingested fluid is in fact cooled by removal of heat from the backface of the wall, and in the process an added cooling load of $(q_c - q_r)$ is incurred. It would require less refrigeration if the interior surface were insulated and the ingested fluid cooled at a location removed from the wall.

The problem of suction applied to a high-energy flow for boundary-layer control purposes has a solution of engineering significance if the backface is taken to be insulated. The constants of integration in Eq. (6) of Ref. 1 are then evaluated subject to the boundary conditions

$$\xi = 0,\ \hat{T} = \hat{t} = 1; \quad \xi = 1,\ \hat{T}' = 0$$

resulting in the solution

$$\hat{T} = \hat{t} = 1 \text{ for } 0 \leq \xi \leq 1$$

That is, the temperature distributions are uniform throughout the wall, and further, from Eq. (1), $q_c = q_r$ as would be expected.

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Reference

[1] Libby, P. A., "Temperature Distribution in a Porous Surface Involving Either Suction or Injection," *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1206–1208.

Reply by Author to A. F. Mills and R. B. Landis

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THE criticism of Mills and Landis is certainly well founded; we agree that the conclusion in Ref. 1 is not supported by the analysis there because of our disregard of a condition of zero net heat flux on the inner wall, a physically significant condition for our purposes. Fortunately, for other recent work which we have based on the use of suction in high-energy flows and for our confidence in simple physical concepts such as energy conservation it is the analysis that is in error, i.e., our major conclusion regarding suction as a source of coolant is correct and is supported by analysis if carried out properly.

The simple concept we refer to is as follows: If steady-state energy conservation is applied to the gas from $x = 0^-$ to $x \to \infty$ (refer to Ref. 1 for notation) and if there is no radiative flux at $x \to \infty$, i.e., if we take the emissivity of the inner surface to be zero, then conservation of energy applied to the gas implies that

$$\rho v c_p(t_0 - t_c) = -q_c + q_r \tag{1}$$

One solution of this equation is that insisted on by Mills and Landis, i.e., $t_0 = t_c$, $q_c = q_r$, but consider an experiment where $\rho v c_p$, t_0, q_c, q_r are all specified; are we to believe that Eq. (1) does not apply and does not determine t_c?

To support our belief in Eq. (1), we correct our previous analysis by imposing a condition of zero heat flux at the inner surface $x = L$. This necessitates consideration of the conductivity of the gas λ_g since we add another boundary condition to those imposed previously. If we restrict our attention to the case of suction $\rho v > 0$,† it is readily clear that for $x > L$, $t \equiv t_c$ and we retain our inner boundary condition $t(L) = t_c$ even though we include conductivity of the gas.

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† Our results for injection should also be corrected but since Mills and Landis commented only on the case of suction we put this matter aside for the time being.